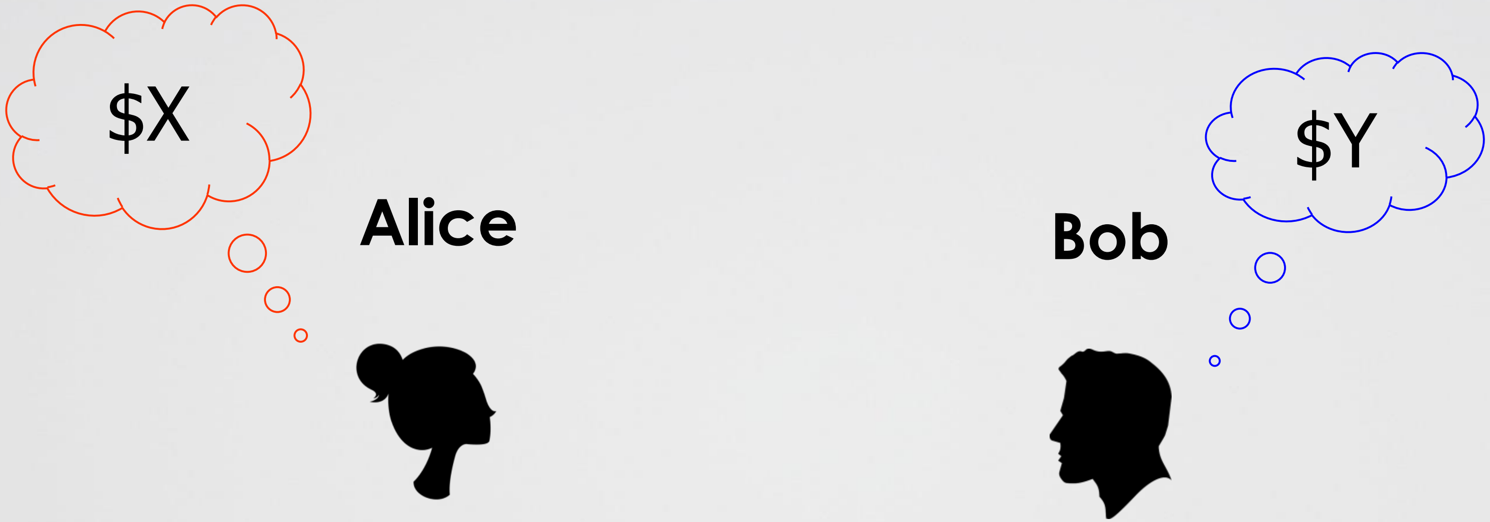
MPC beyond the Generic Model and Private Intersection Analytics

Benny Pinkas, Bar-Ilan University

(with many coauthors)

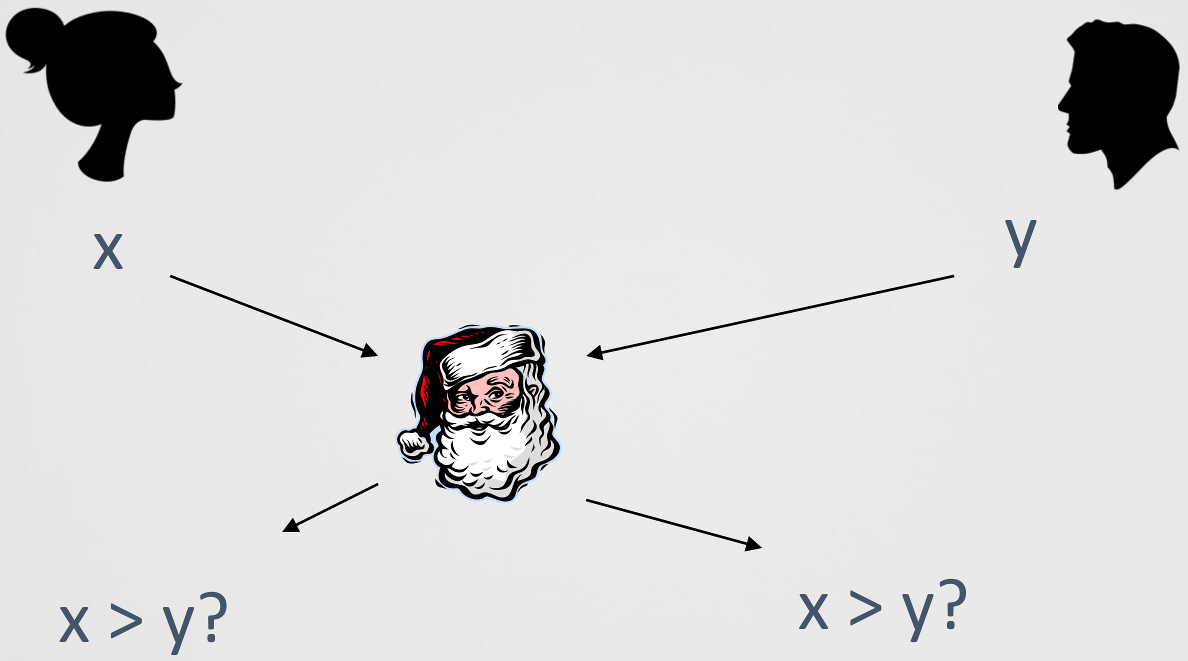
The millionaires' problem (Yao, 1982)



- Want to find out if X > Y
- But leak no other information! (even to each other)
- Standard crypto tools (encryption) do not help in this case!

The millionaires' Problem

As if…



The millionaires' Problem

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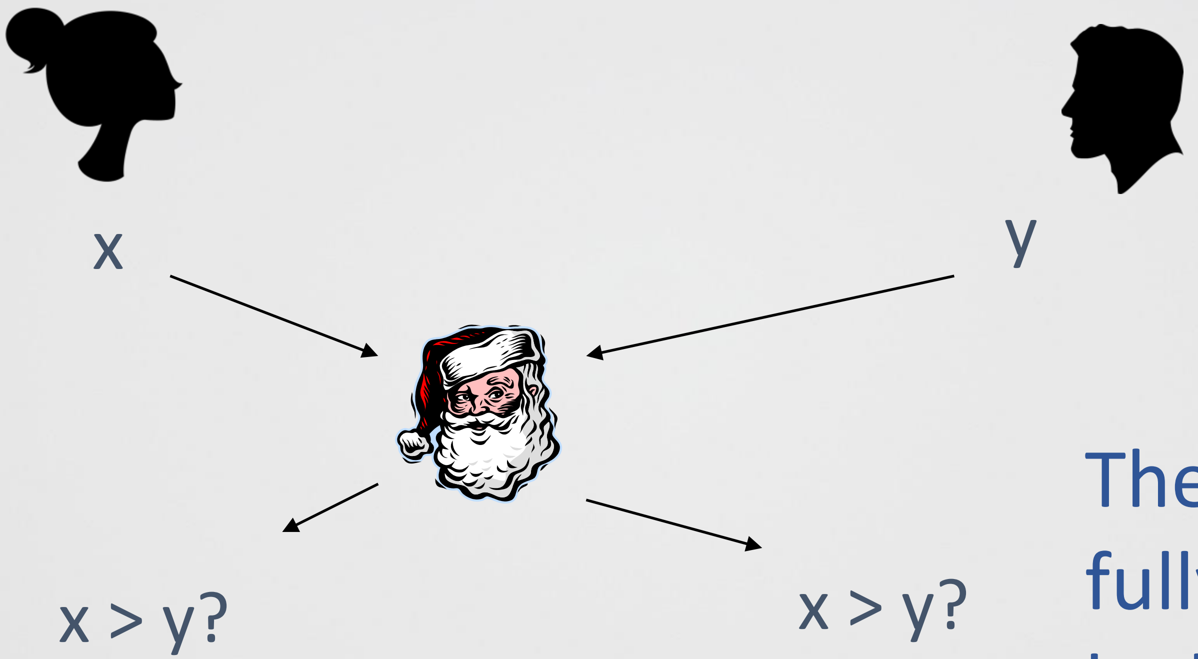


The trusted party is fully secure and leaks no information!

Research on MPC

- MPC started as a curious mental game/challenge
  - Millionaires problem, poker over the phone,…
- MPC research was theoretical for many years (1982 - 1998)
  - Focused on feasibility results and not on implementation
- This type of basic research is important

Applications of the millionaires problem?

Trading

- Alice: I want to sell x stocks, for a minimum price of P_{ASK}
- Bob: I want to buy y stocks, for a maximum price of P_{BID}

- Output:
  - If $P_{ASK} > P_{BID}$ then output “no deal”
  - Otherwise output “you can trade $\min(x,y)$ stocks”

- Auctions and bidding

Defining security

Desired Security of MPC

Input: x y

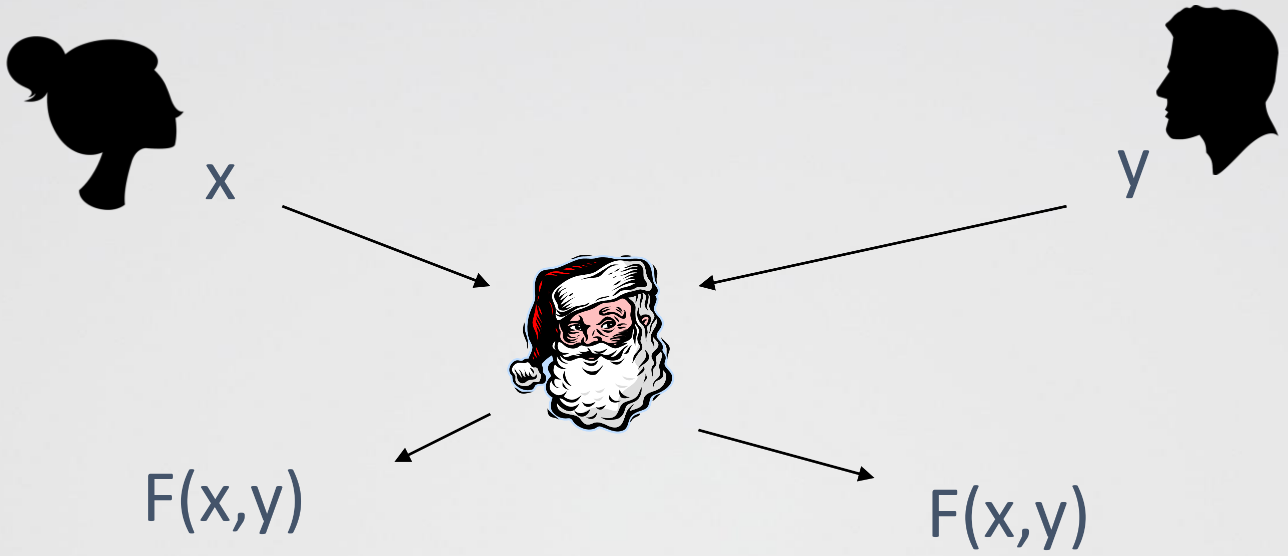
Output: F(x,y) *and nothing else*

As if…

x y

F(x,y) F(x,y)

Does this model make sense?



- We cannot hope for more privacy
- Does the trusted party scenario make sense?
  - Are the parties motivated to submit their true inputs?
  - Can they tolerate the disclosure of F(x,y)?
- If so, can implement the scenario without a trusted party

MPC is about a distributed implementation of a trusted party

Instead of

MPC does **not** tell you which function is safe to compute,

do

…

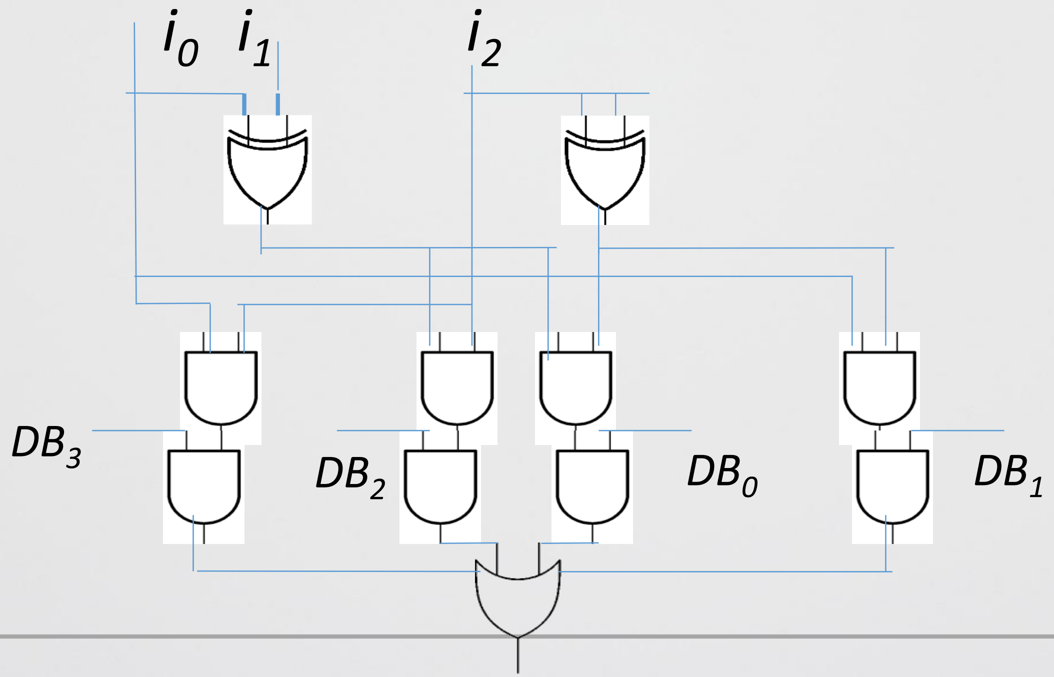
learn F(x,y) *and nothing else*

only how to compute it without trusted party

Implementing Secure Computation

Generic secure computation (Yao, 1982)

- Can be used to securely compute **any** function
- Considered theoretical, until the Fairplay system [MNPS04]
- Based on representing the function as a **Boolean circuit**



Implementing generic secure computation

- A lot of very smart optimizations in recent years
- Actual performance depends on circuit size, and on
    - setting
    - security requirements
    - preprocessing
    - engineering
  - Secure computation of AES: between 1μs to 1sec per block
  - We can easily handle circuits with 10^6 - 10^9 gates

MPC Beyond Generic Computation

What we know

1. Efficient circuit → efficient MPC protocol
2. Function with polynomial run time → circuit of polynomial size
3. (1) + (2) → if we can efficiently compute a function then we can also run an MPC computing it

- Overhead of MPC depends on the circuit representation

Examples

- Alice has integer x, Bob has integer y
  - Computing x+y, x-y
  - Computing whether x>y, max(x,y)
  - Computing $x \cdot y$, x^y
- X, Y are sets
  - Computing $X \cap Y$
  - Computing median(X,Y)
- X is an array, y is an index
  - Computing X[y]

Examples

- Alice has integer x, Bob has integer y
  - Computing $x+y$, $x-y$ (easy)
  - Computing whether $x>y$, $\max(x,y)$ (easy)
  - Computing $x \cdot y$, x^y (less easy)
- X, Y are sets
  - Computing $X \cap Y$ (less easy)
  - Computing median(X,Y) (less easy)
- X is an array, y is an index
  - Computing X[y] (not easy)

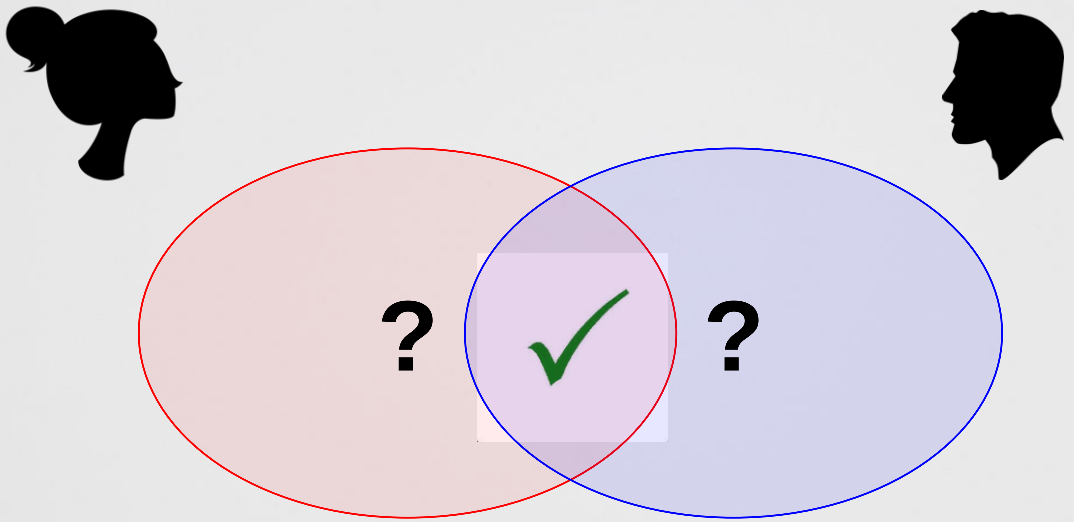
Specific vs. Generic Protocols

- Sometimes we can design a specific protocol for a specific problem, which will be more efficient than a generic, circuit-based protocol
- Still, it is preferable to use a circuit-based generic protocol
- We'll now show how to get the best of both worlds

Private Set Intersection (PSI) and Analytics

PSI Background, and Why Circuit-Based PSI?

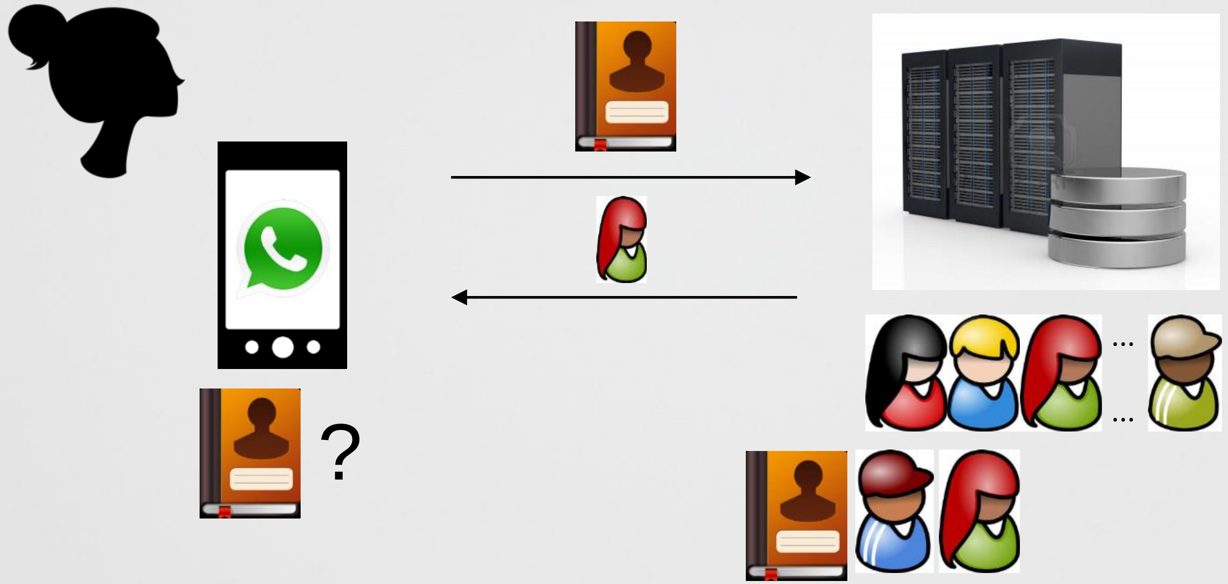
Private Set Intersection (PSI)



Applications of PSI

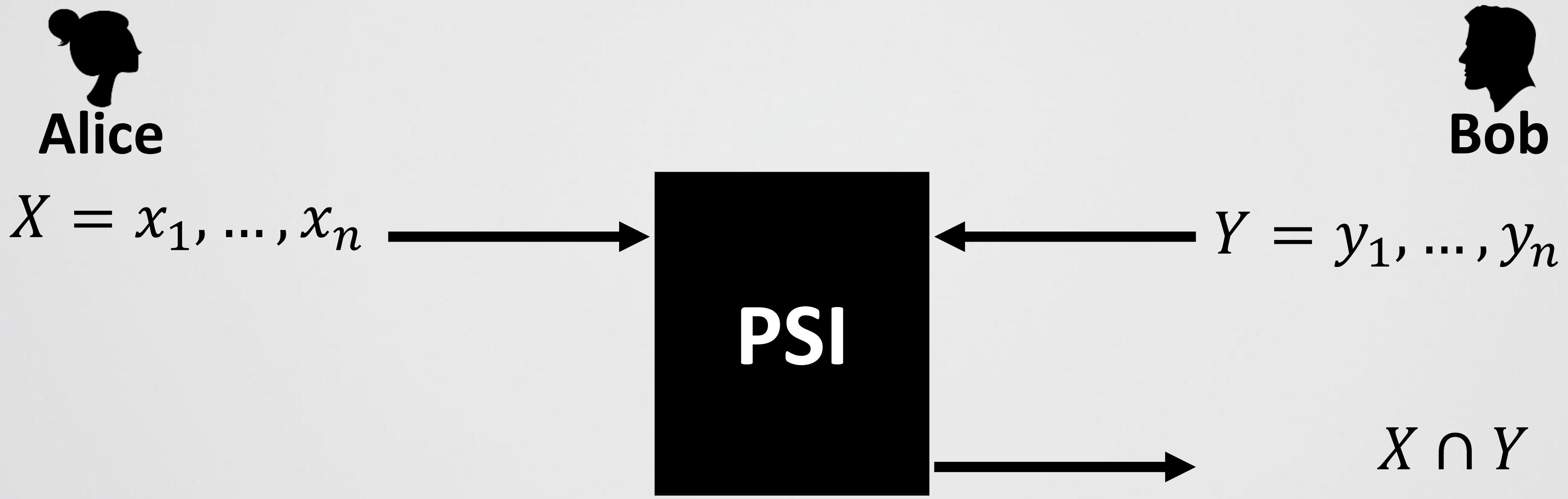
- **Information sharing**, e.g., intersection of threat information
- **Matching**, e.g., testing compatibility of different properties (preferences, genomes…)
- **Join** DB operations
- **Analytics:** $\Pr(A / B) = \Pr(A \cap B) / \Pr(B)$
- **Identifying mutual contacts** (Signal app)
- **Computing ad conversion rates** (Google)

Application: Common Contacts



Private Set Intersection (PSI)

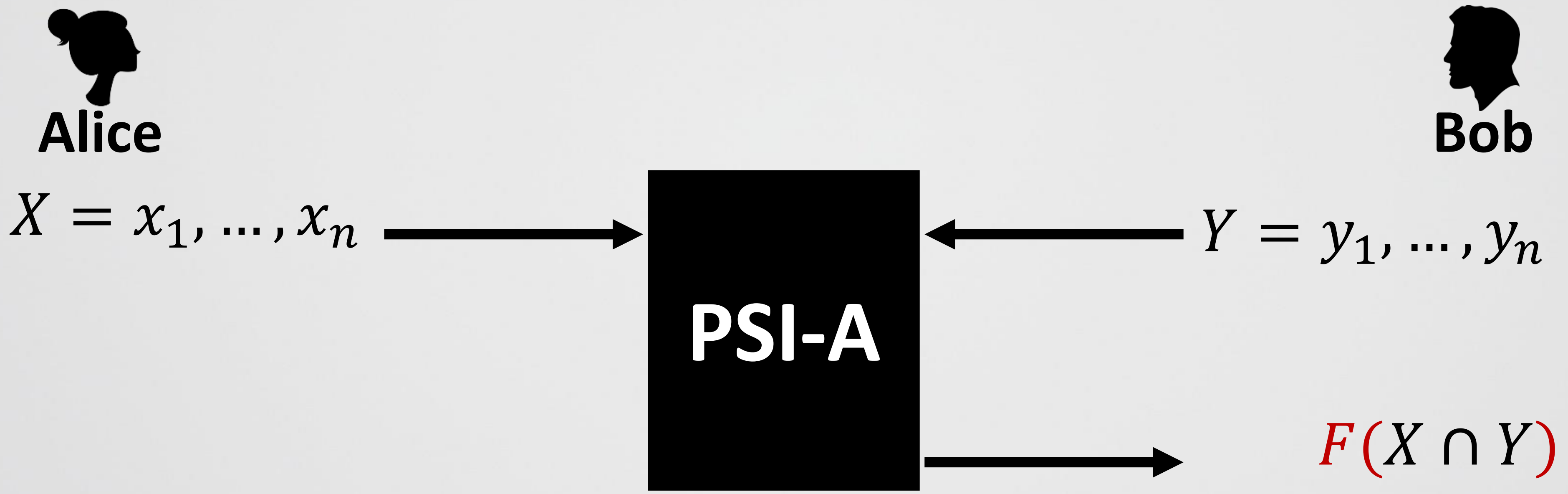
Definition



Private Set Intersection (PSI) - Analytics

Definition

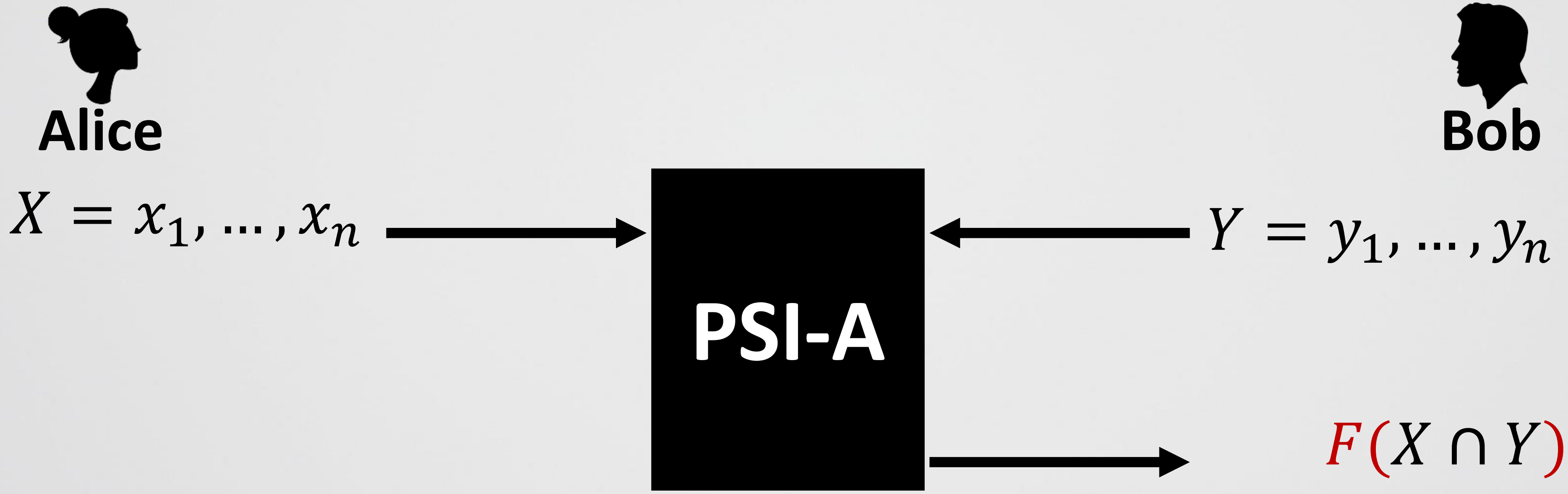
1. Post processing function F.   E.g. $F = |X \cap Y|$



Private Set Intersection (PSI) - Analytics

Definition

1. Post processing function F.   E.g. $F = |X \cap Y|$
2. Items are associated with payloads (aka. PSI with data-transfer)



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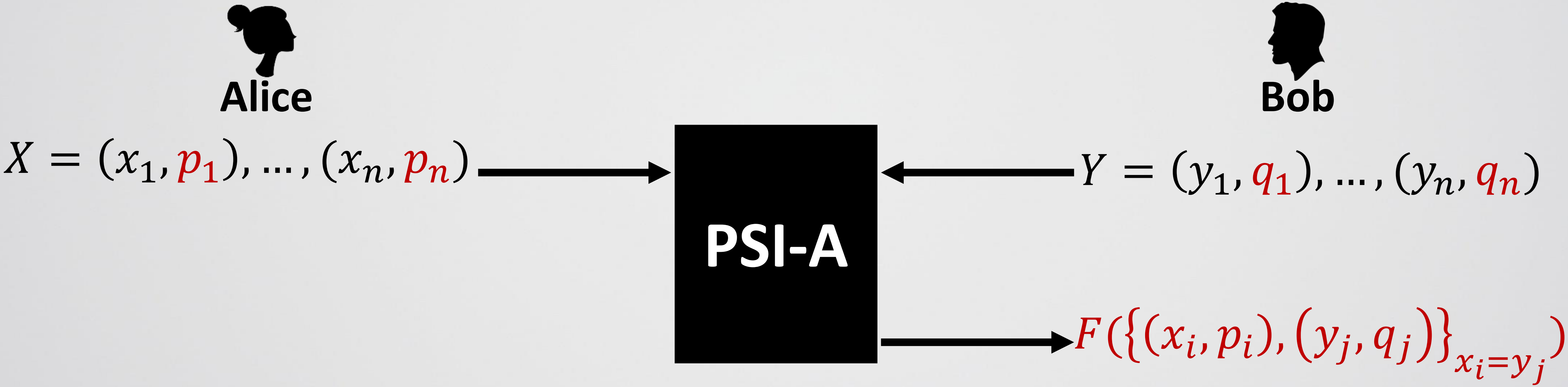

Private Set Intersection (PSI) - Analytics

Applications

Private Intersection-Sum Protocol with Applications to Attributing Aggregate Ad Conversions

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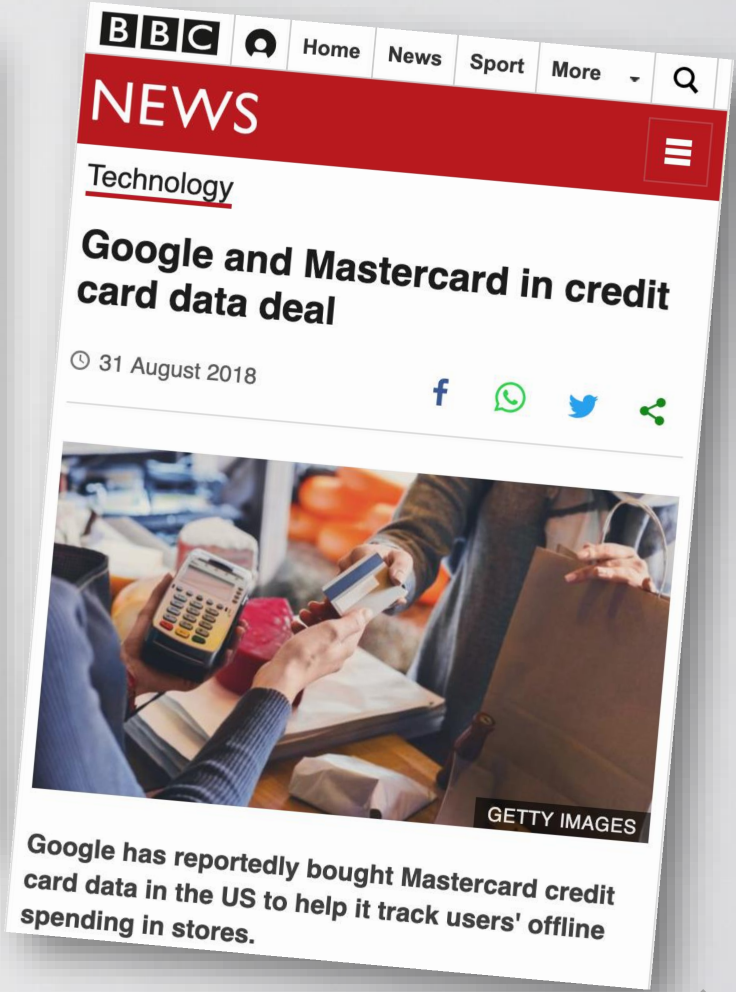
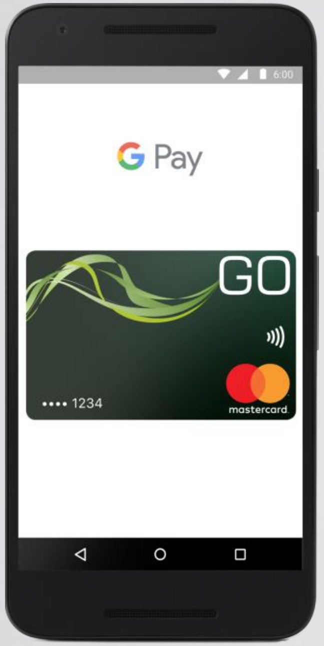
July 31, 2017



Private Set Intersection (PSI) - **Analytics**

Applications: **Online Ads to Offline Purchase Conversion**

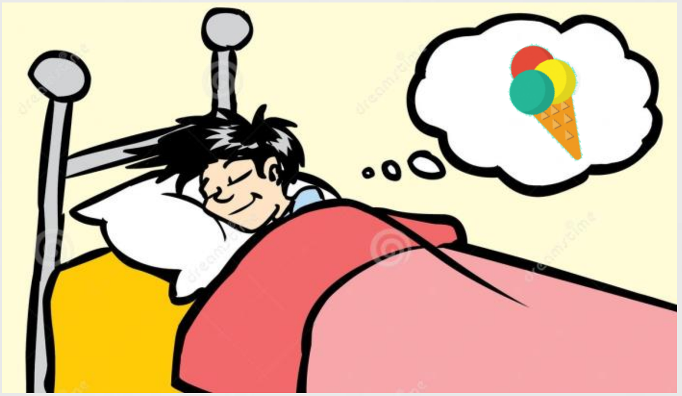


Register

1

Watch online ad

2



Dream about it

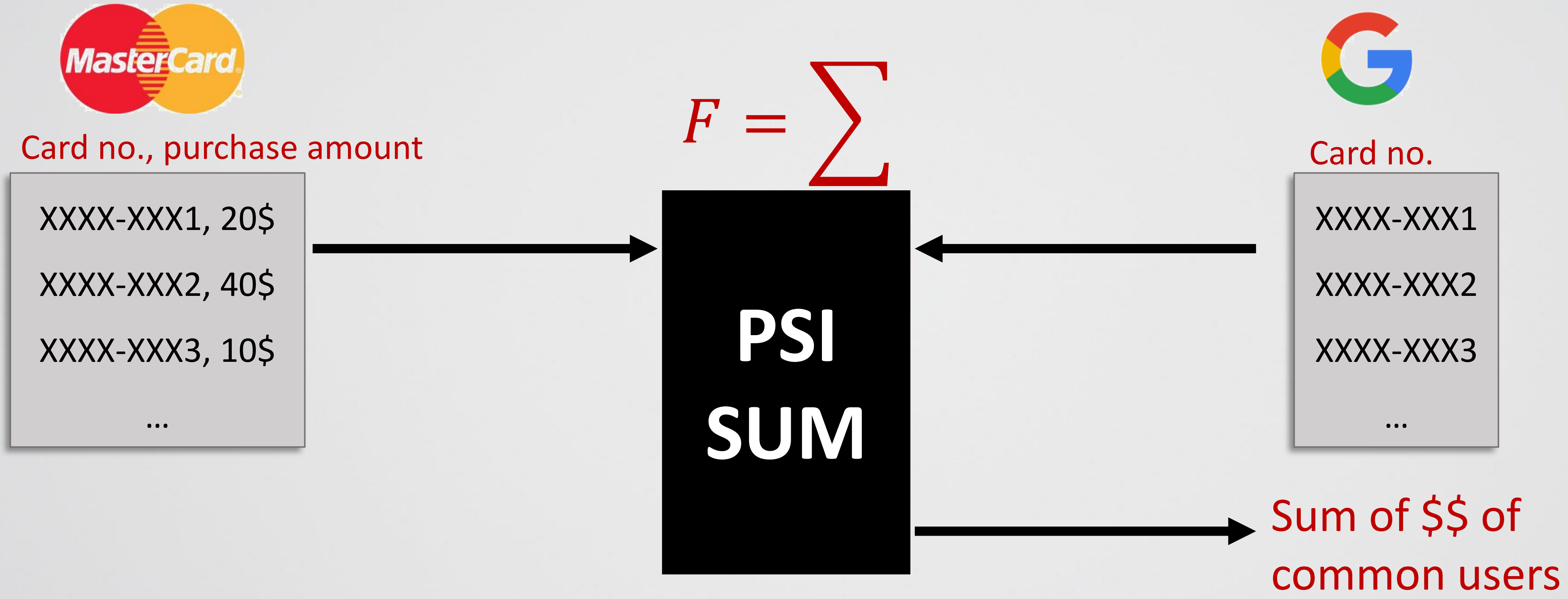
3

Buy offline

4

Private Set Intersection (PSI) - Analytics

Applications: Online Ads to Offline Purchase Conversion



Implementing PSI

Private Set Intersection (PSI)

Naïve solution

Insecure when items have low entropy

Alice

Bob

$H(x_1), \dots, H(x_n)$

Compare to
$H(y_1), \dots, H(y_n)$

Public-key based Protocols for PSI

(for example, based on the Diffie-Hellman assumption)

PSI based on Diffie-Hellman

- [S80, M86, HFH99, AES03]:

(ignore the details)

α 

$x_1,\ldots,x_n$

 β

$y_1,\ldots,y_n$

$(H(x_1))^\alpha,\ldots,(H(x_n))^\alpha \longrightarrow$

$\longleftarrow (H(y_1))^\beta,\ldots,(H(y_n))^\beta$

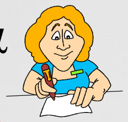
$\longleftarrow ((H(x_1))^\alpha)^\beta,\ldots,((H(x_n))^\alpha)^\beta$

$((H(y_1))^\beta)^\alpha,\ldots,((H(y_n))^\beta)^\alpha$

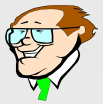
Compares the two lists

PSI based on Diffie-Hellman

- [S80, M86, HFH99, AES03]:

α 

$x_1,\ldots,x_n$

 β

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$((H(y_1))^\beta)^\alpha,\ldots,((H(y_n))^\beta)^\alpha$

Compares the two lists

- Simple to understand ☺
- Simple to implement ☺
- Can be based on elliptic-curve crypto ☺
- Minimal communication ☺ but a lot of computation ☹

More recent PSI constructions [PSZ1,PSSZ15,KKRT16]

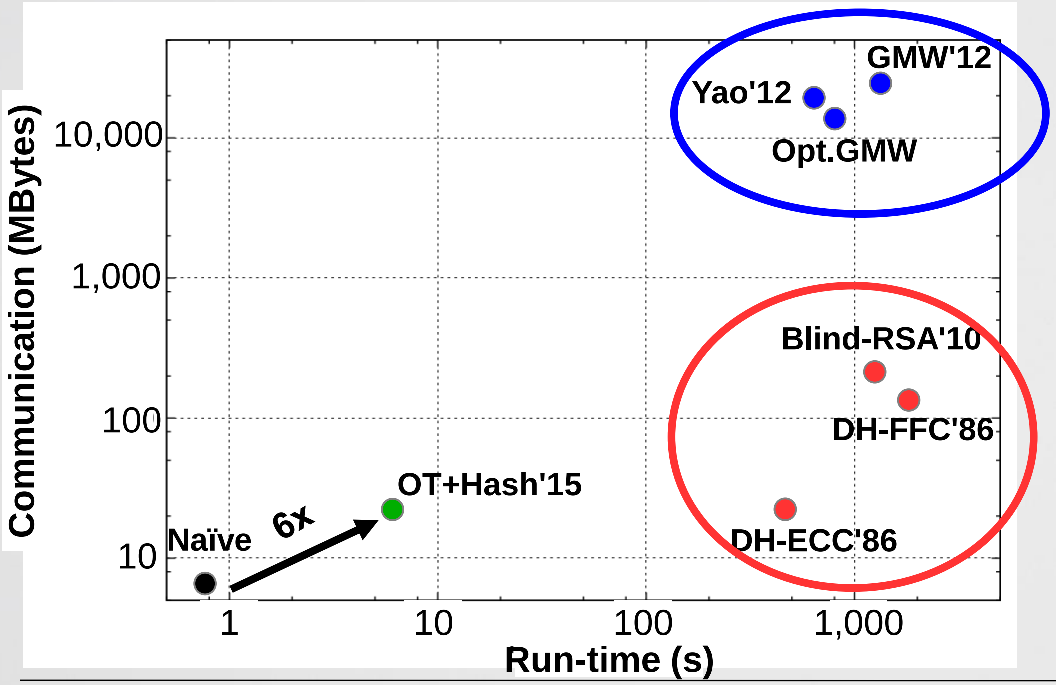
- PSI is “equivalent” to oblivious transfer

- Oblivious transfer extension [IKNP04] is very fast, and can enable very efficient PSI

- Used different hashing ideas to dramatically reduce the overhead of PSI

Performance Classification of PSI protocols [PSZ]

- PSI on $n = 2^{18}$ elements of $s=32$-bit length for 128-bit security on Gbit LAN

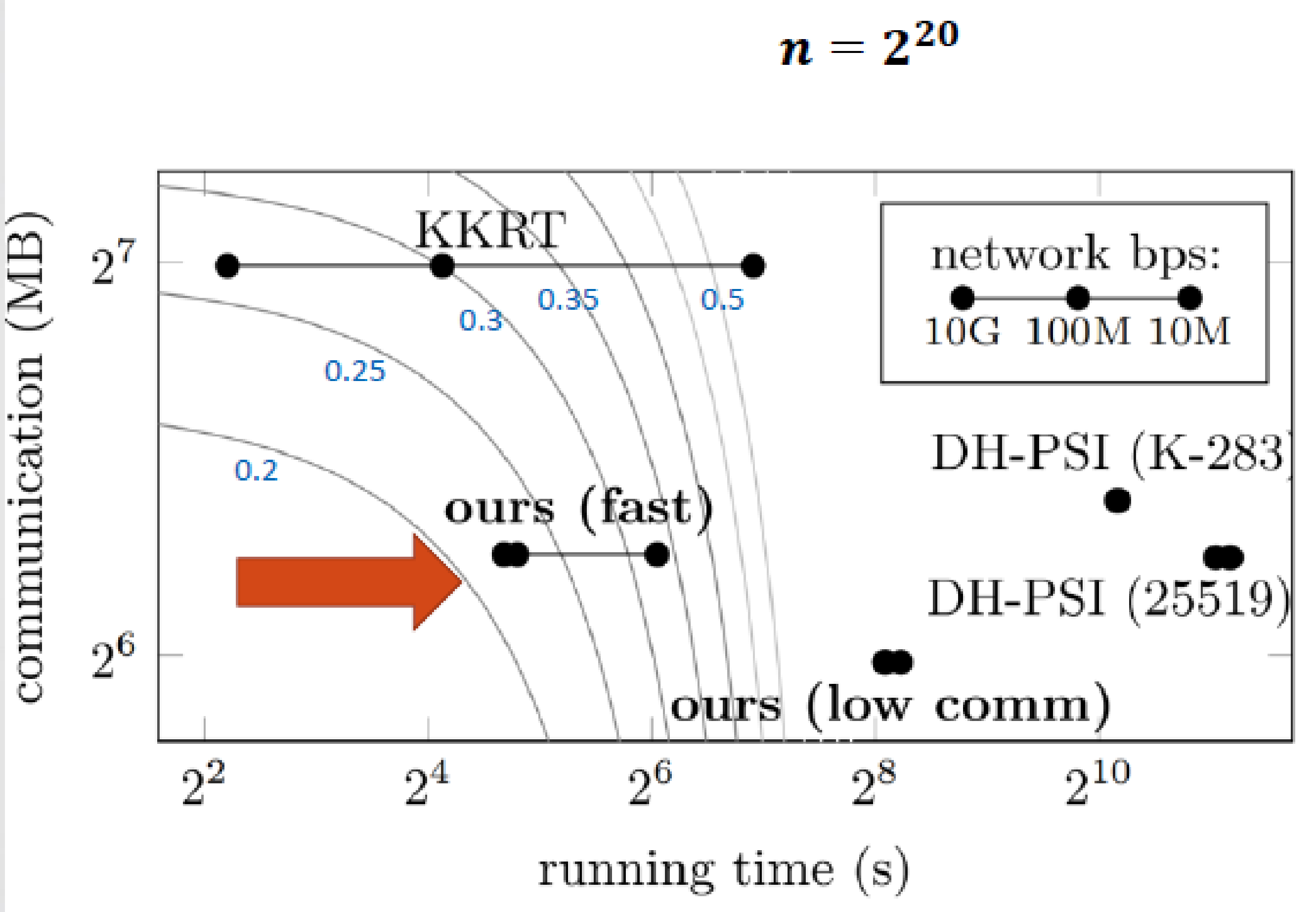


Circuit-Based (PSI analytics):
- high run-time & communication, but easily extensible to arbitrary functions

PK-Based: (starting from [S80,M86])
- high run-time
+ best communication

OT-Based:
[PSZ15,PSSZ16,KKRT16] good communication and run-time

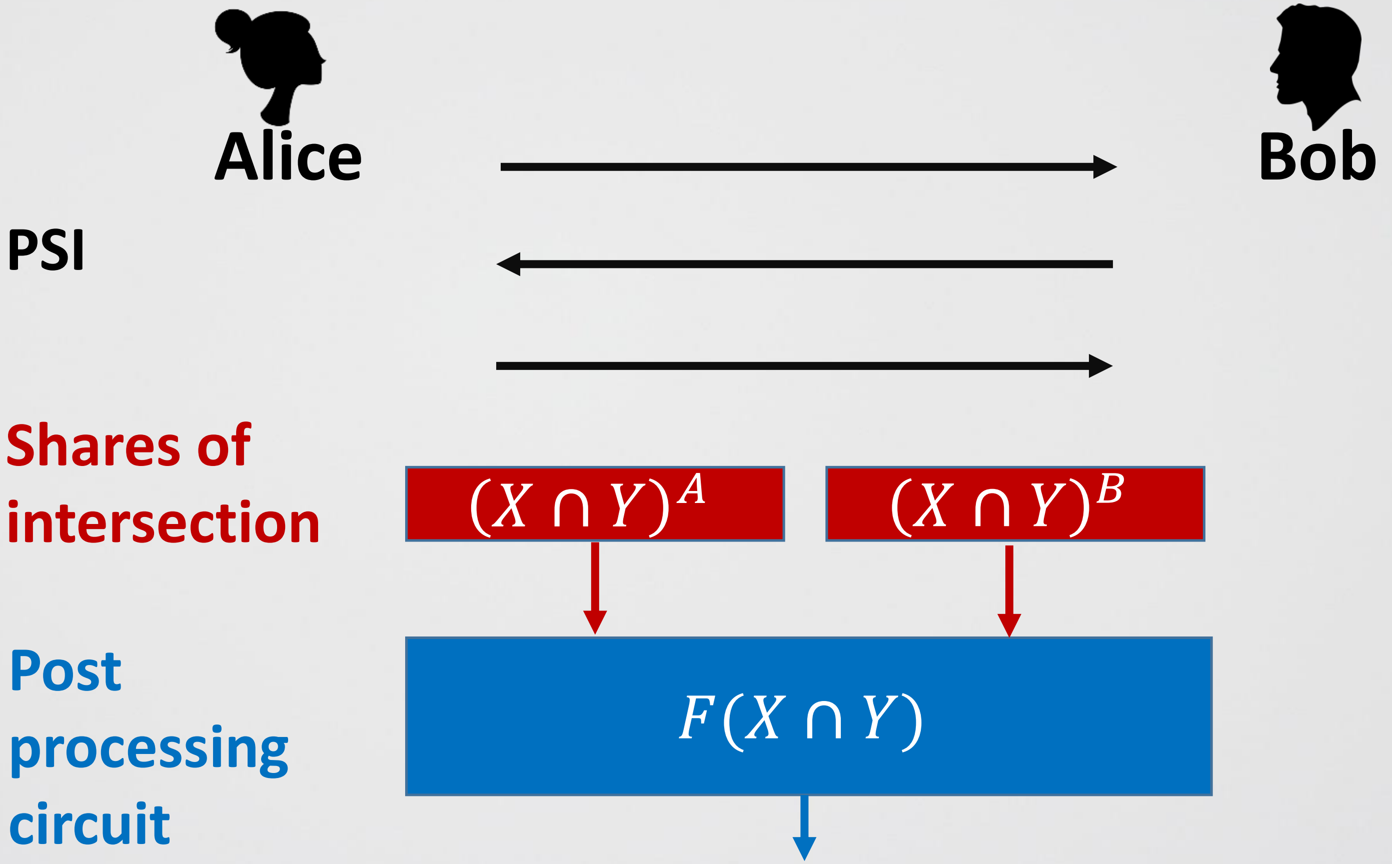
SpOT PSI (Crypto 2019 [PRTY])



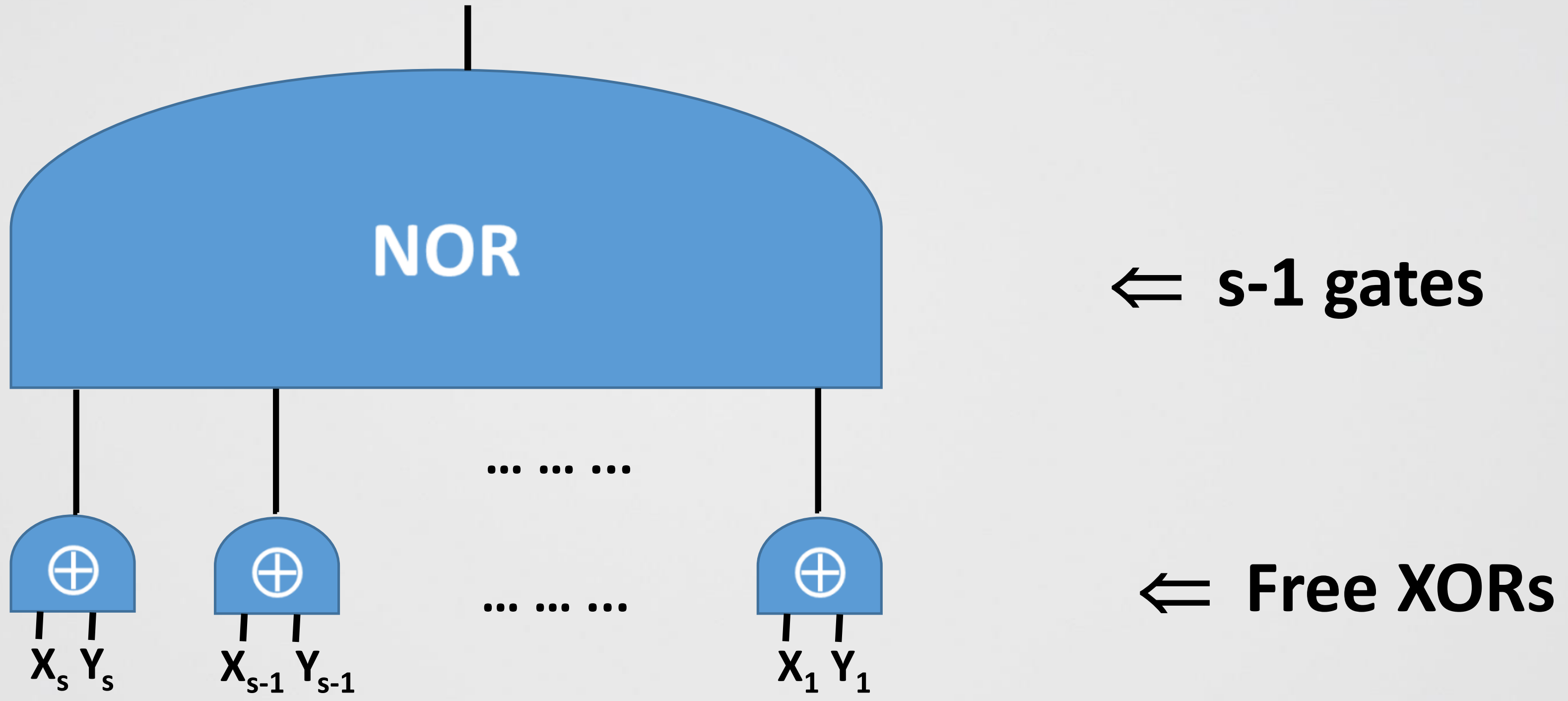
Private Set Intersection (PSI) - Analytics

Also known as: **Circuit-Based PSI**

- Immediate security for any F
- Modularity
- Existing code base



A circuit comparing two s-bit values (x=y?)



The Algorithmic Challenge

- Goal: Find the smallest circuit for computing the intersection
  - Alice and Bob **can prepare** their inputs
  - Circuit must **not** depend on data!

- Any symmetric function of the intersection could be then added
  - E.g., the **size** of the intersection, or whether size is greater than some **threshold**, potentially after adding noise to ensure **differential privacy**

Known circuit-based protocols for PSI

- A naïve circuit for PSI uses n^2 comparisons
- A protocol based on sorting networks – $O(n \log n)$ comparisons [HEK12]
- A protocol based on OT and hashing– $O(n \log n / \log\log n)$ comparisons [PSSZ16]

Known circuit-based protocols for PSI

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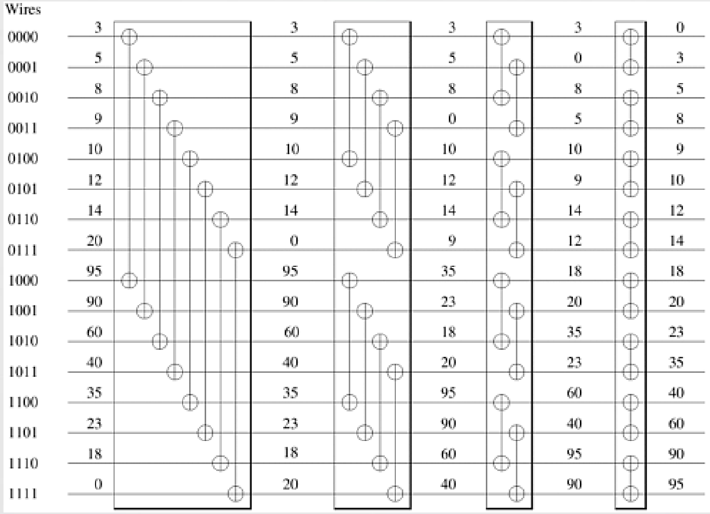
- We reduced the overhead to $O(n)$ [PSTY19]

A circuit based PSI protocol [HEK12]

- A PSI circuit that has three steps
  - Sort: merge two sorted lists using a bitonic merging network [Bat68]. Uses $n\log(2n)$ comparisons.

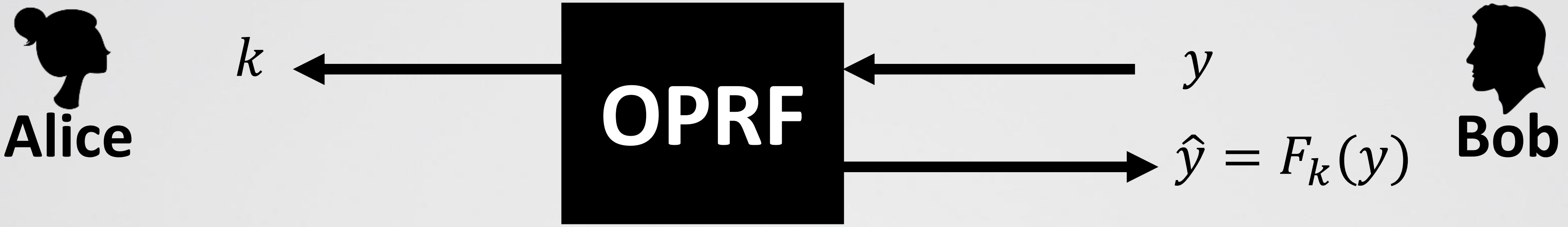


A circuit based PSI protocol [HEK12]

- A circuit that has three steps
  - Sort: Merge two sorted lists using a bitonic merging network [Bat68]. Computes the sorted union using $n\log(2n)$ comparisons.
  - Compare: Compare adjacent items. Uses $2n$ equality checks.
  - Shuffle: Randomly shuffle results using a Waxman permutation network [W68], using $\sim n\log(n)$ swappings.
  - Overall Computes $O(n\log n)$ comparisons.
    Uses $s\cdot(3n\log n + 4n)$ AND gates. (s is input length)

Private Set Intersection (PSI)

Main tool: **Oblivious PRF (OPRF) [FIPR05]**



- To compare $x_1, \dots, x_n$ to y Alice sends:

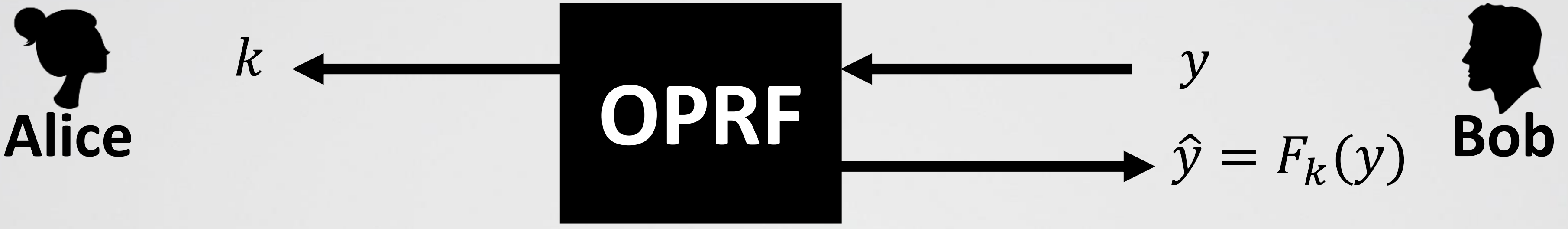
$$\hat{x}_1, \dots, \hat{x}_n = F_k(x_1), \dots F_k(x_n) \longrightarrow$$

If y $\notin \{x_1, \dots, x_n\}$ then it looks random to Bob

$$\hat{x}_1 =? \; \hat{y}$$
$$\hat{x}_2 =? \; \hat{y}$$
$$\dots$$
$$\hat{x}_n =? \; \hat{y}$$

Private Set Intersection (PSI) - Analytics

Main tool: **Oblivious PRF (OPRF) [FIPR05]**

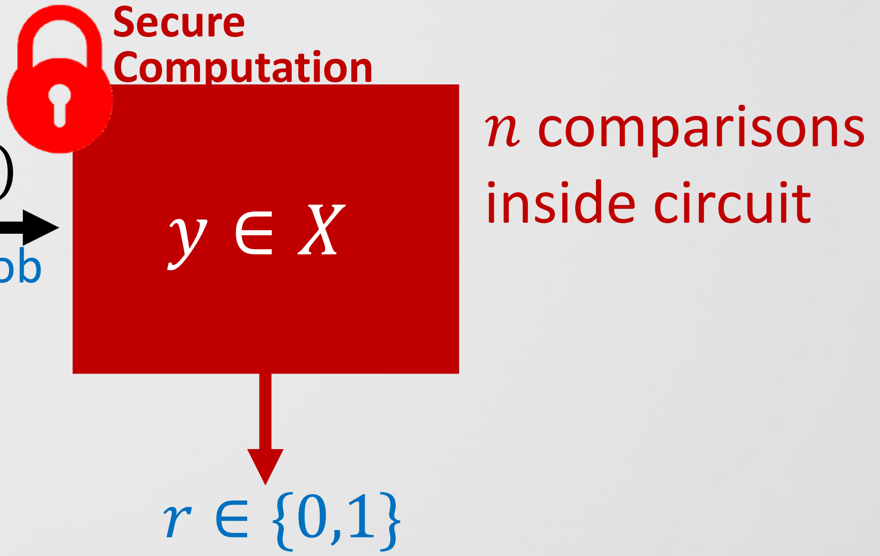


- To compare $x_1, \dots, x_n$ to y Alice sends:

$$\hat{x}_1, \dots, \hat{x}_n = F_k(x_1), \dots F_k(x_n)$$

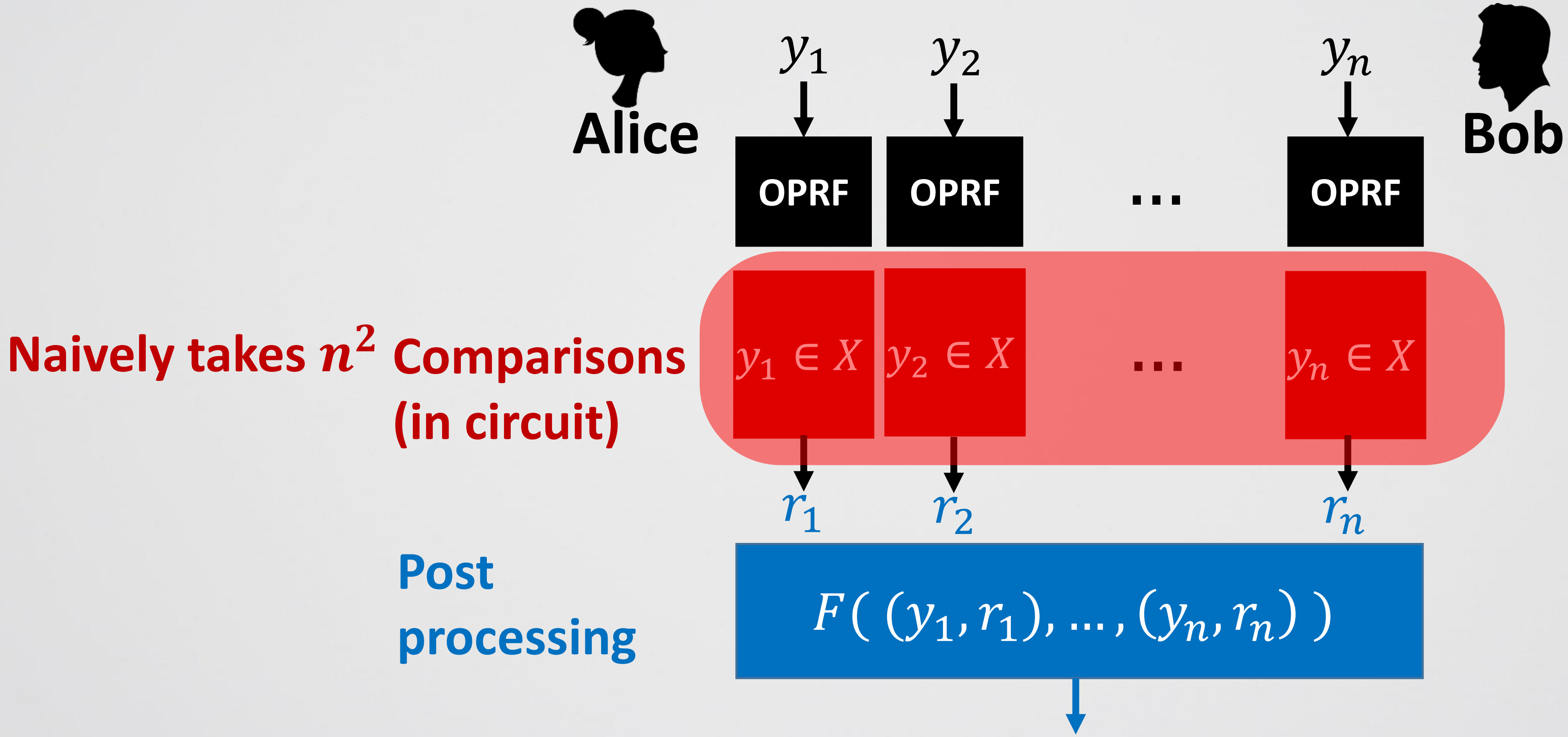
If y $\notin \{x_1, \dots, x_n\}$ then it looks random to Bob

- To compute a function – compare in a 2PC circuit:



Private Set Intersection (PSI) - Analytics

Protocol Overview



Results [PSTY19]

	PSI Analytics	PSI
Asymptotic	First linear-communication protocol (in **OT-hybrid** model and assuming **correlation-robust hash function**)	
Concrete	vs. [PSWW18] 10x less communication 3-6x faster	vs. DH-based 10-20% less communication 7x faster !
Functionality	Payload from both parties	vs. [KKRT16] 40-50% less communication 2-6x slower in LAN (10 Gbps) 2x faster in WAN (10 Mbps)

Cheapest in $ (always)

PSI + Hashing

Map to bins

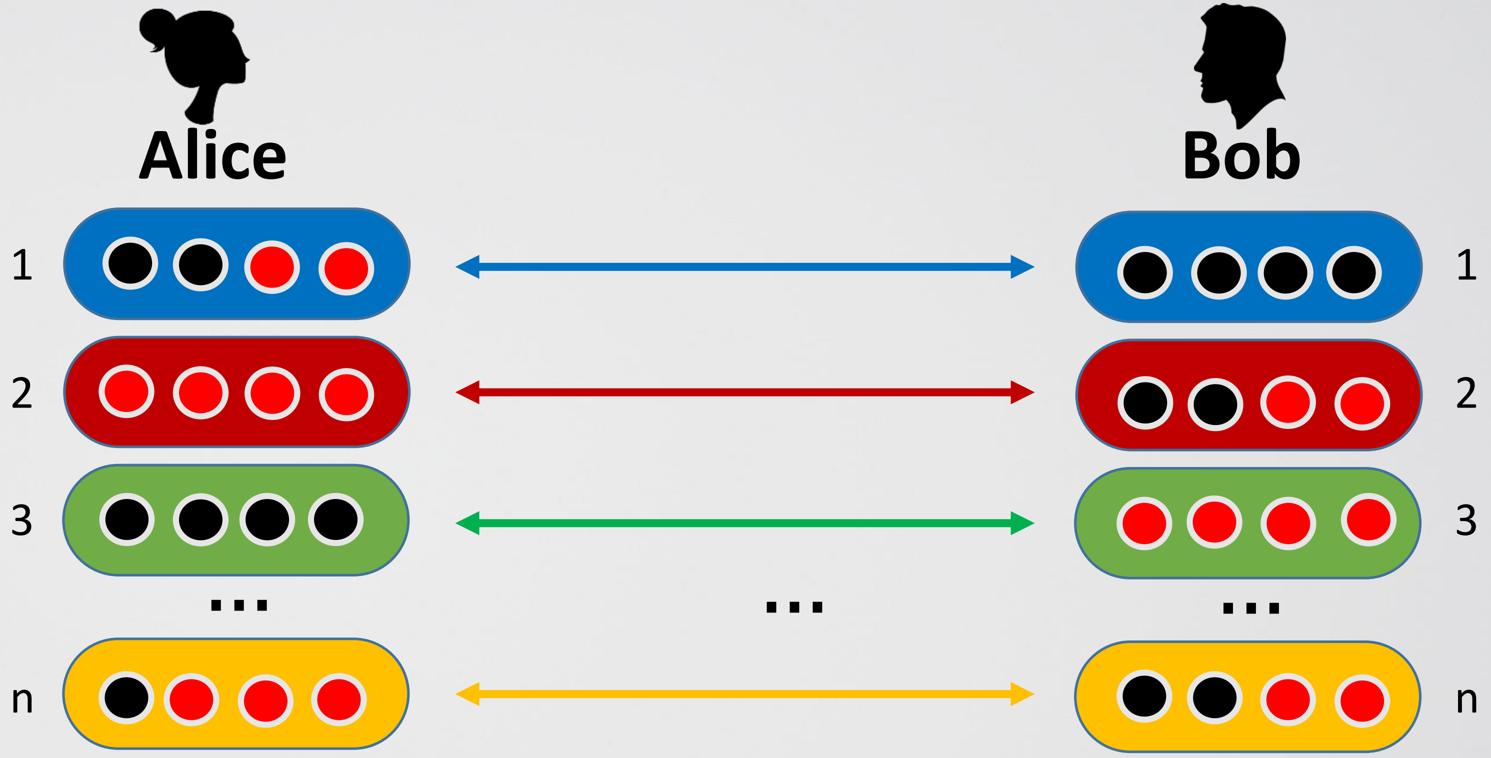
- h: item $\rightarrow$ bin
- Map n items to n bins
  - Some bins may have multiple items

PSI + Hashing

Map to bins

- Use a public h: item $\rightarrow$ bin
- Map n items to n bins
  - Some bins may have multiple items
- Perform bin-wise PSI
- Must hide # items per bin: $(< M = O(\log n)$ w.h.p)
  - Pad bins
- # in-circuit comparisons:

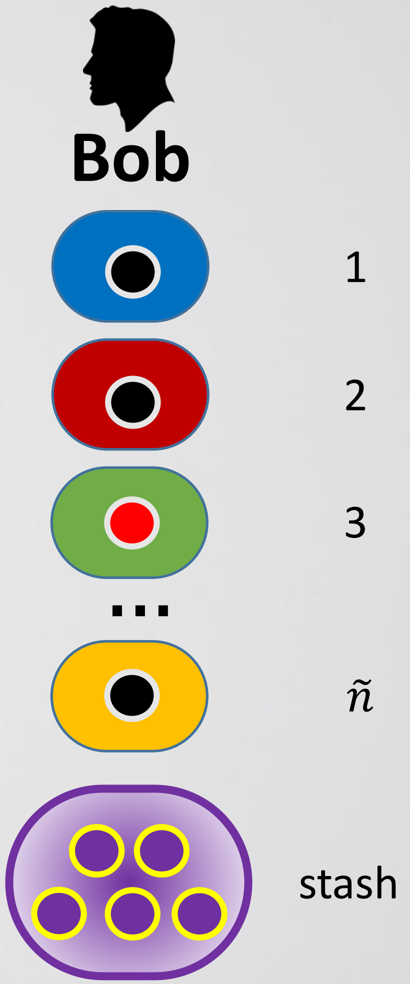
$$n \cdot M^2 = O(n \log^2 n)$$



Using 2 Hash Functions (Cuckoo hashing [PR,KMW])

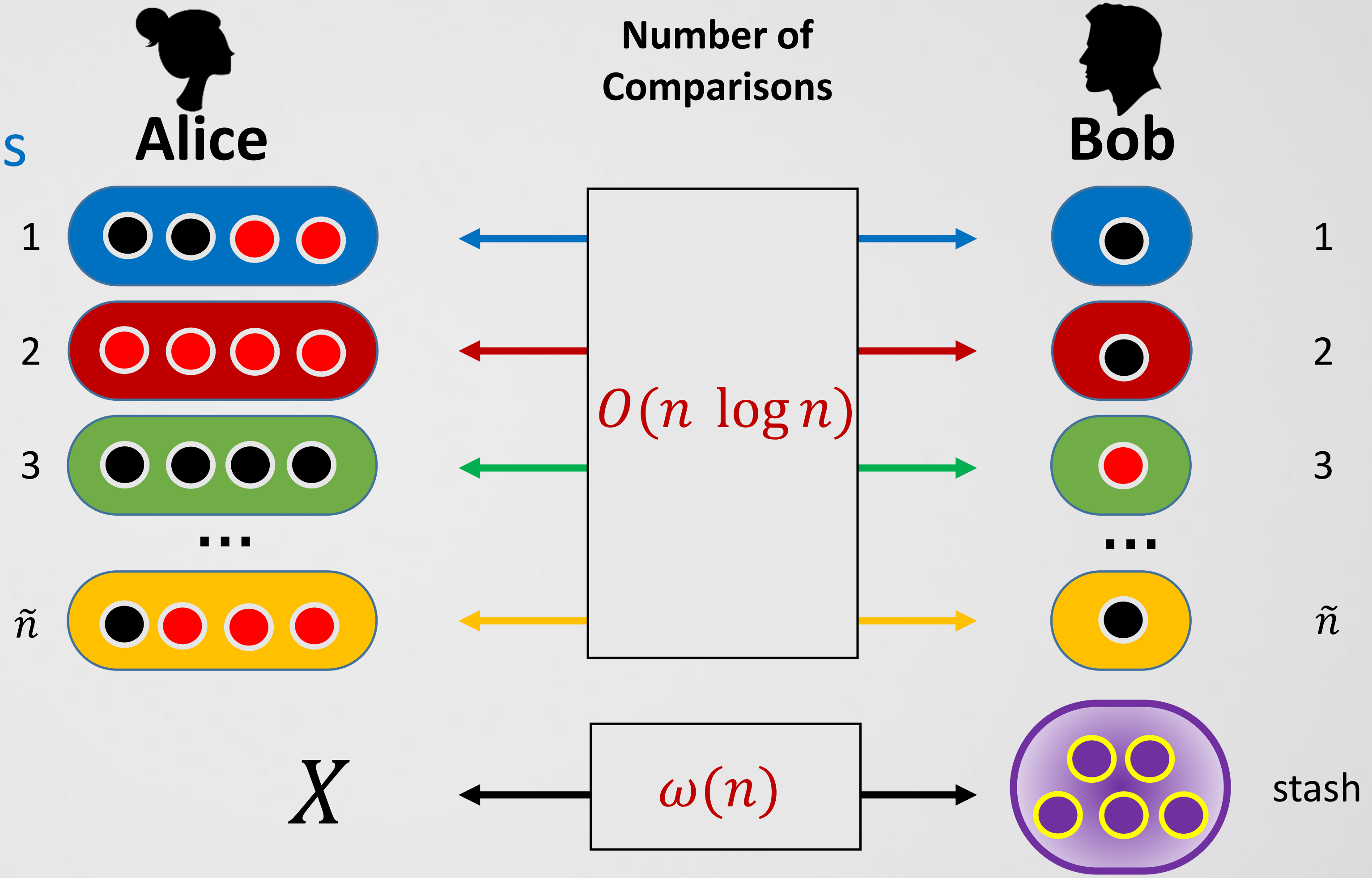
- h_1, h_2: item $\rightarrow$ bin
- Map n items to $(2+\epsilon)n$ bins
- Each bin can store at most one item!

- Succeeds with very high probability
- If we also have a stash of size s, all items *x* can be mapped to either *$h_1(x), h_2(x)$* or the stash, except with probability *$n^{-(s+1)}$*.



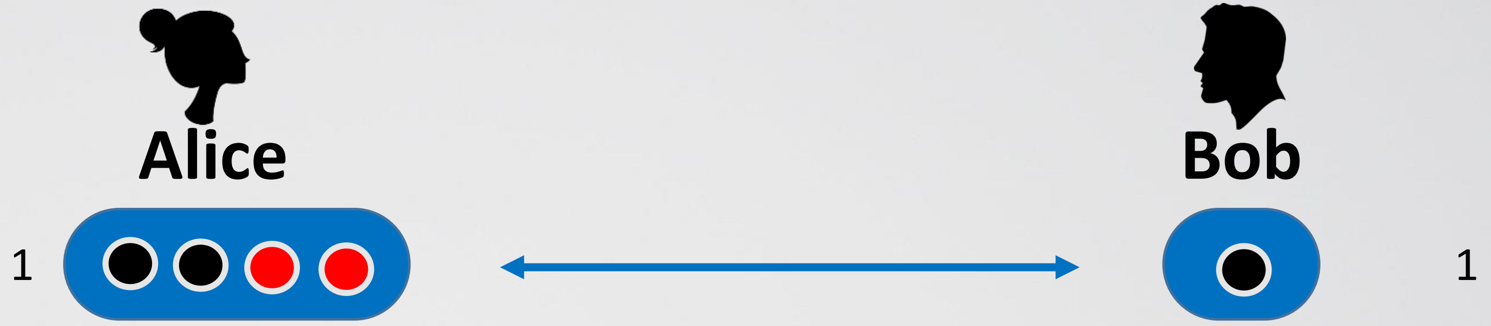
The Power of Using 2 Hash Functions (Cuckoo)

- h_1, h_2: item $\rightarrow$ bin
- Map n items to $(2+\epsilon)n$ bins
  - Alice – simple hashing
    - $x \rightarrow h_1(x)$ **and** $h_2(x)$
    - Max < $M = O(\log n)$
  - Bob – Cuckoo hashing
    - $y \rightarrow h_1(y)$, **or** $h_2(y)$
    - Max ≤ 1
- **Caveat:** stash size $\omega(1)$



Our Protocol

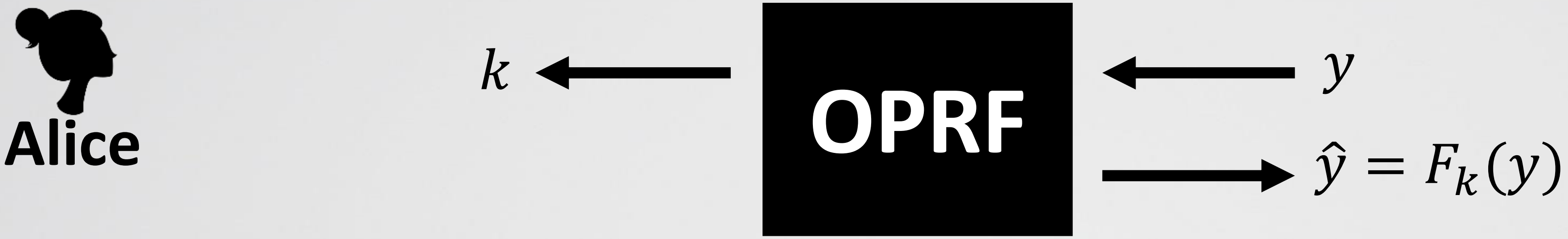
1st step



- **Recall:** each bin contributes $O(\log n)$ comparisons to the circuit
- We transform it to **only 1 comparison** using **Oblivious *Programmable* PRF**

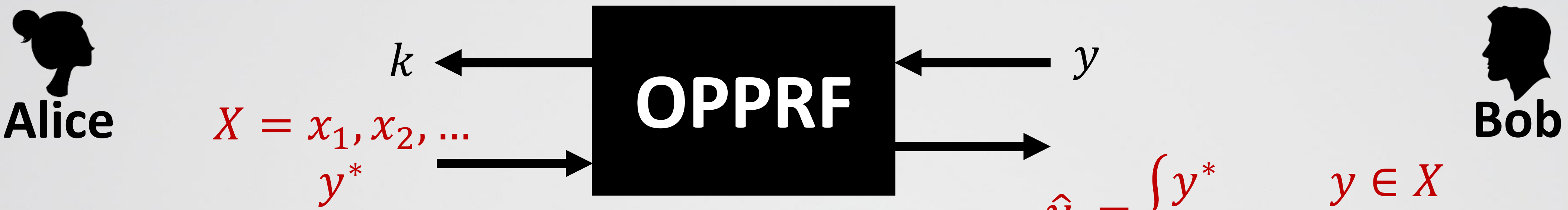
Our Protocol

Oblivious *Programmable* PRF (OPPRF) [KMPRT17]



Our Protocol

Oblivious *Programmable* PRF (OPPRF) [KMPRT17]



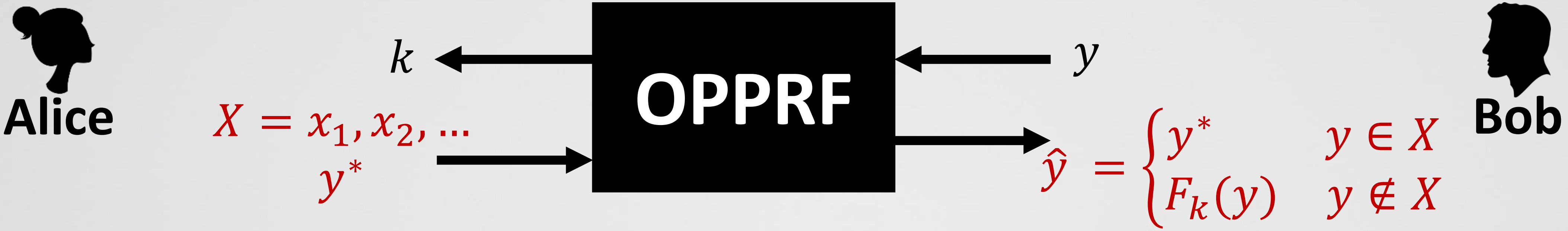
$$\hat{y} = \begin{cases} y^* & y \in X \\ F_k(y) & y \notin X \end{cases}$$

- To compare $x_1, x_2, \dots$ to y Alice sends:

$$\xrightarrow{\quad y^* \quad} \quad y^* =? \, \hat{y}$$

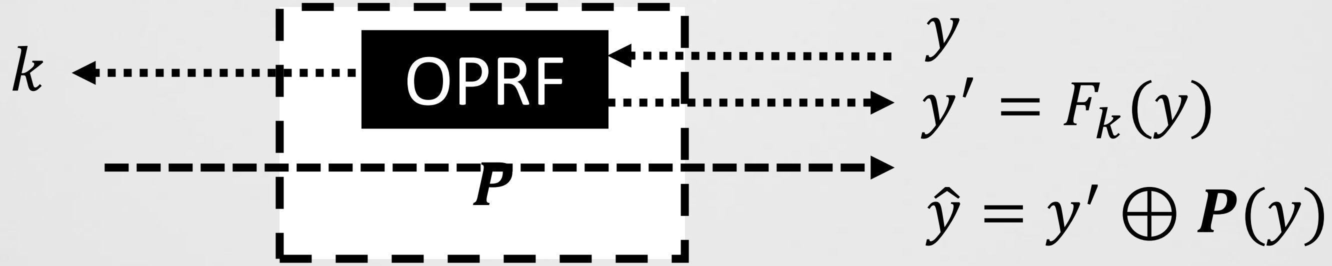
Our Protocol

Implementing Oblivious *Programmable* PRF (OPPRF) [KMPRT17]



$\hat{x}_i = F_k(x_i) \oplus y^*$

Interpolate $\boldsymbol{P}: x_i \to \hat{x}_i$



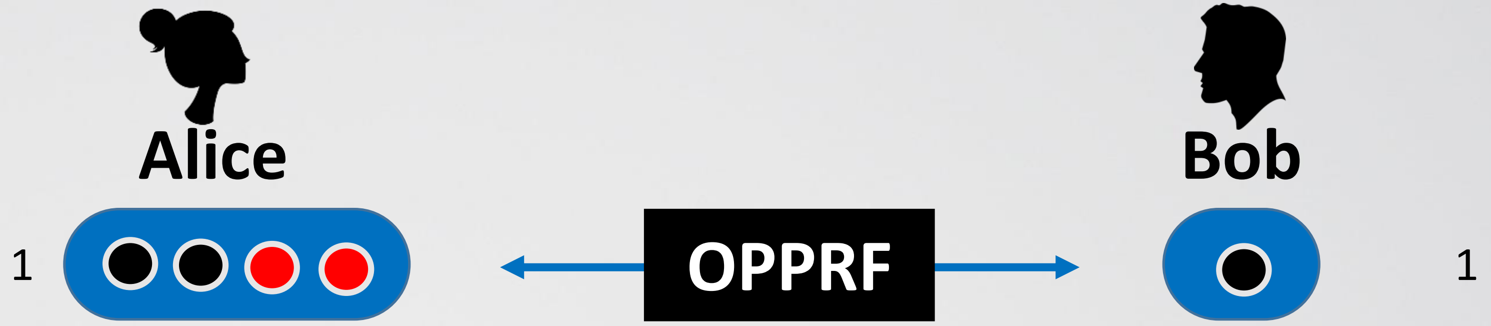
$$y = x_i \Rightarrow \hat{y} = F_k(x_i) \oplus \boldsymbol{P}(x_i) = F_k(x_i) \oplus F_k(x_i) \oplus y^* = y^*$$

$$y \neq x_i \Rightarrow \hat{y} = F_k(y) \oplus \boldsymbol{P}(y)$$

Our Protocol

1st step: "Programming" the PRF

- Alice "programs" $O(\log n)$ items
- ➔ Single comparison in the secure computation



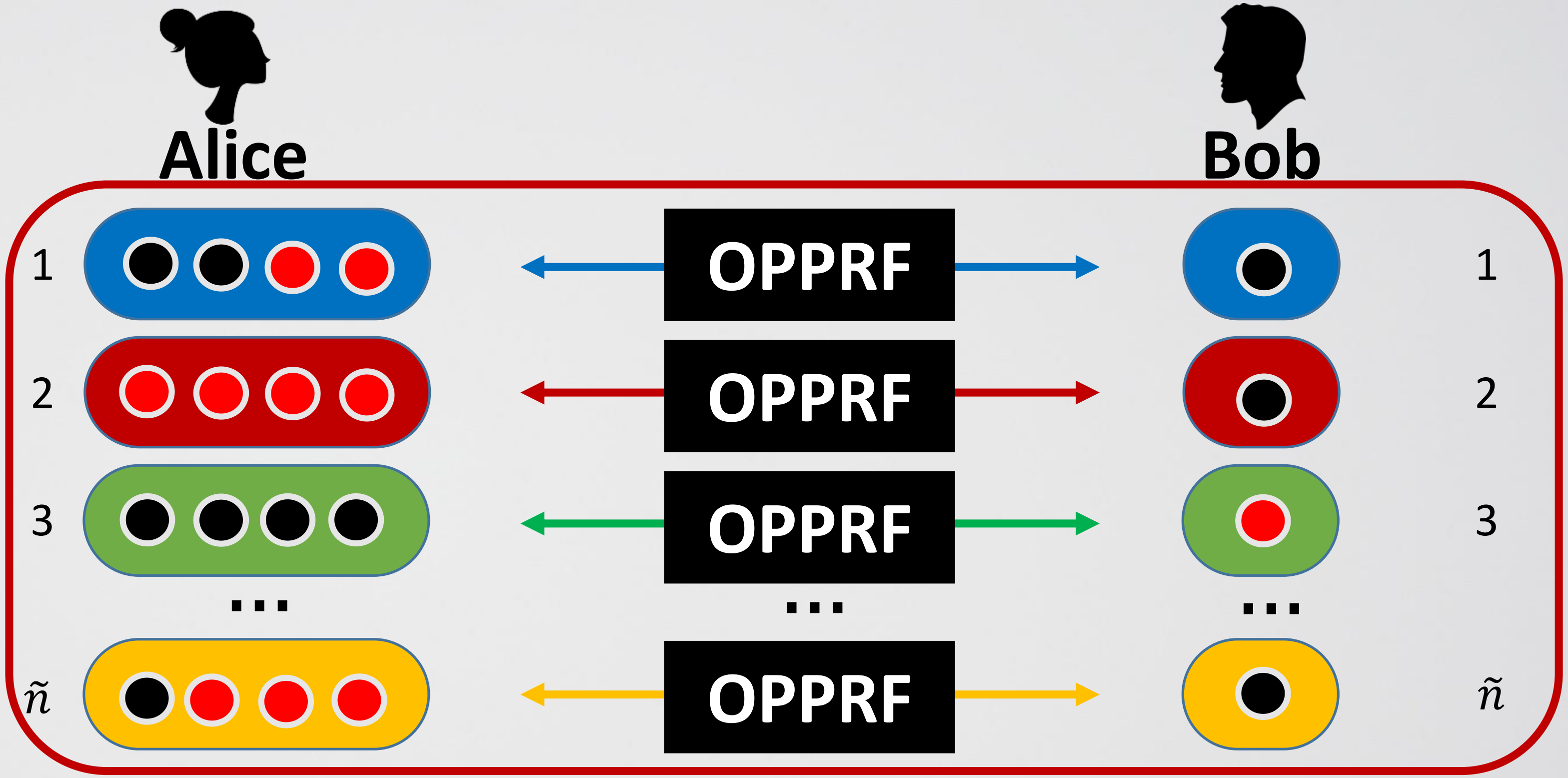
- **Communication of OPPRF:**
  - For each bin, linear in the number of programmed values
  - ➔ communication per bin remains $O(\log n)$

Our Protocol

2nd Step: Batching the OPPRF

- We can "batch" many OPPRFs with comm. $O(n)$
- **Preserving obliviousness without programming padded values!**

$O(n)$



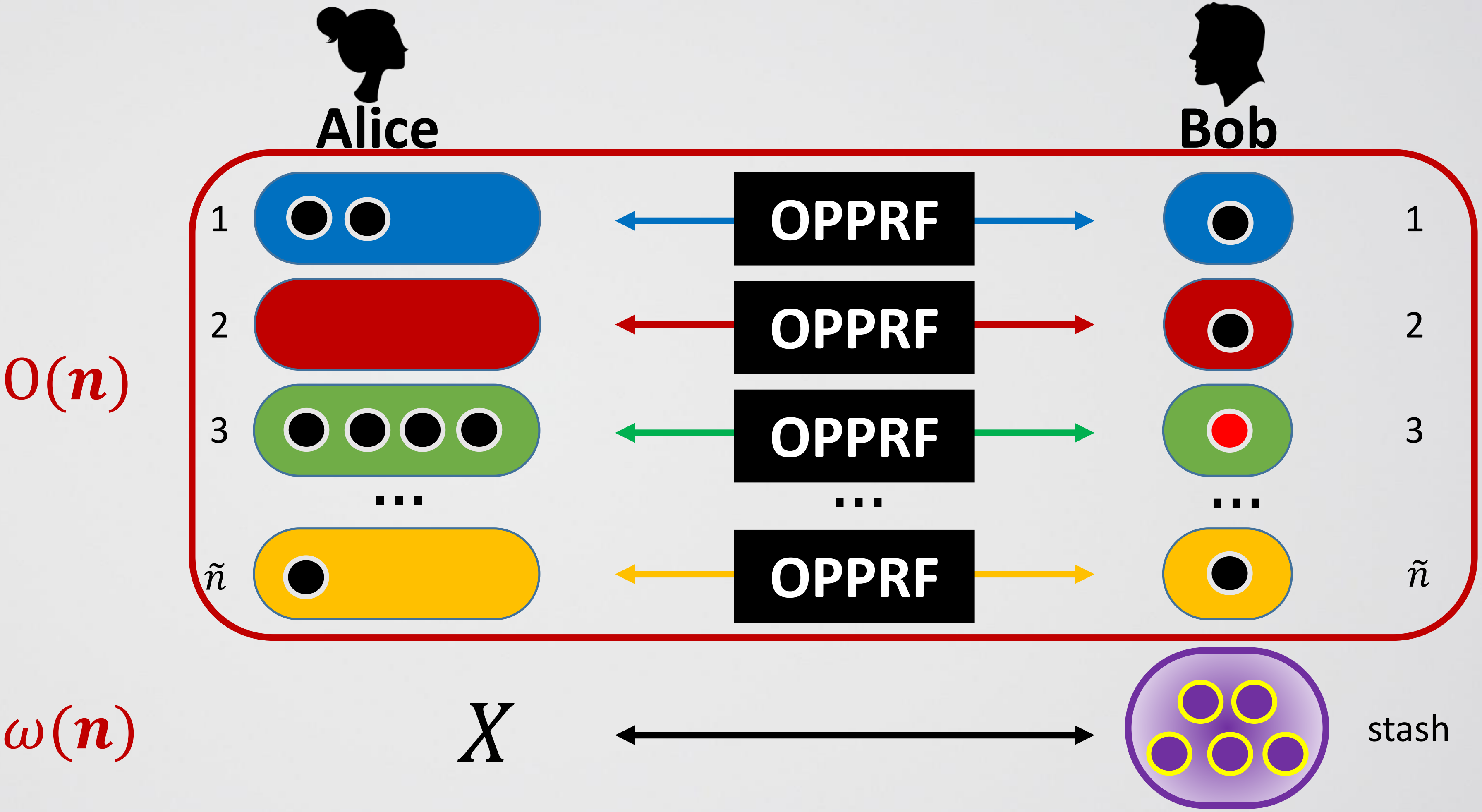
Using high degree polynomials

- Need to interpolate very high degree polynomials over arbitrary points
- Lagrange interpolation is too slow

- **Used FFT to do that with overhead $O(n\log^2 n)$**

Our Protocol

3rd Step: Handling the Stash



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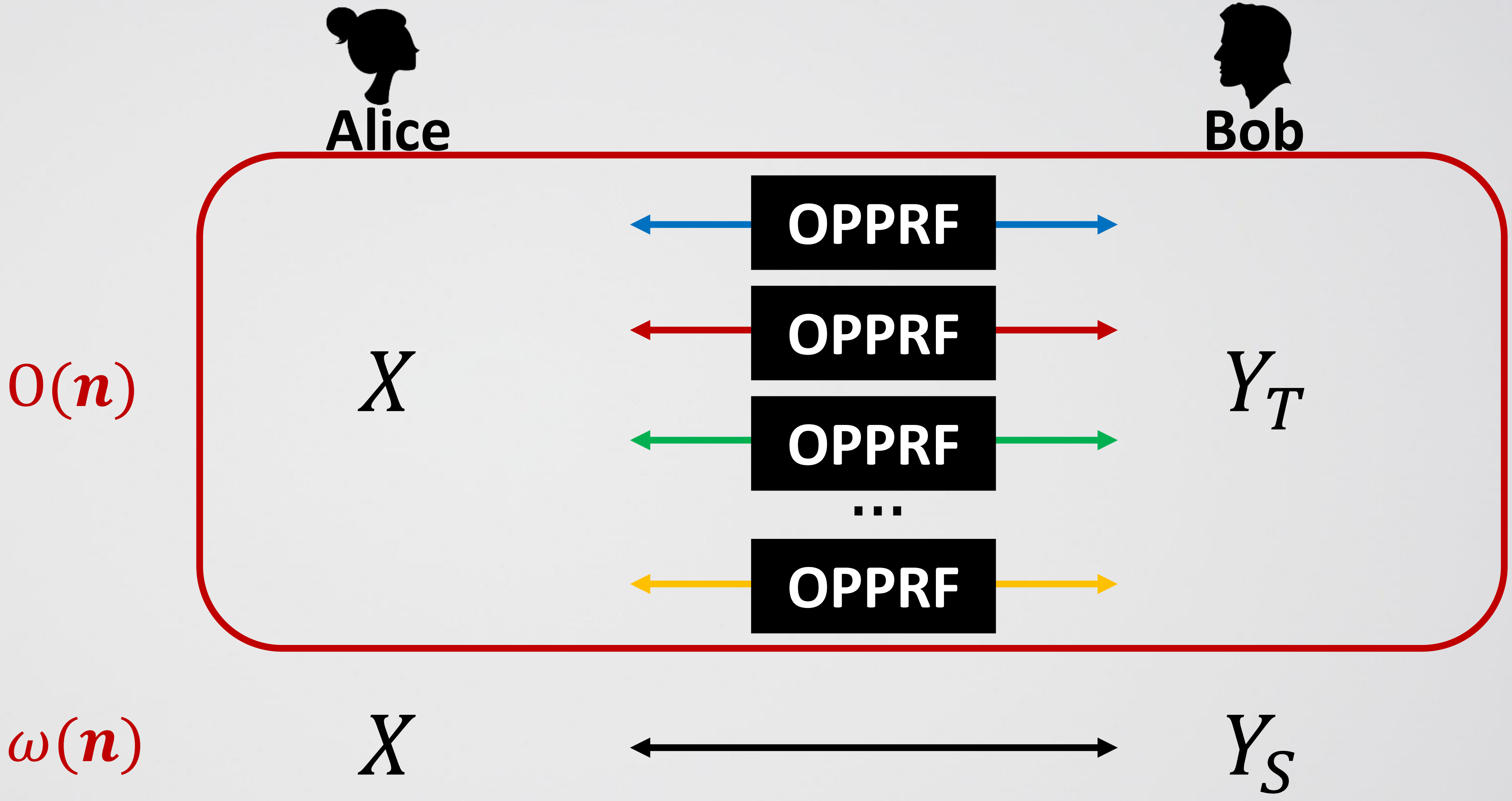
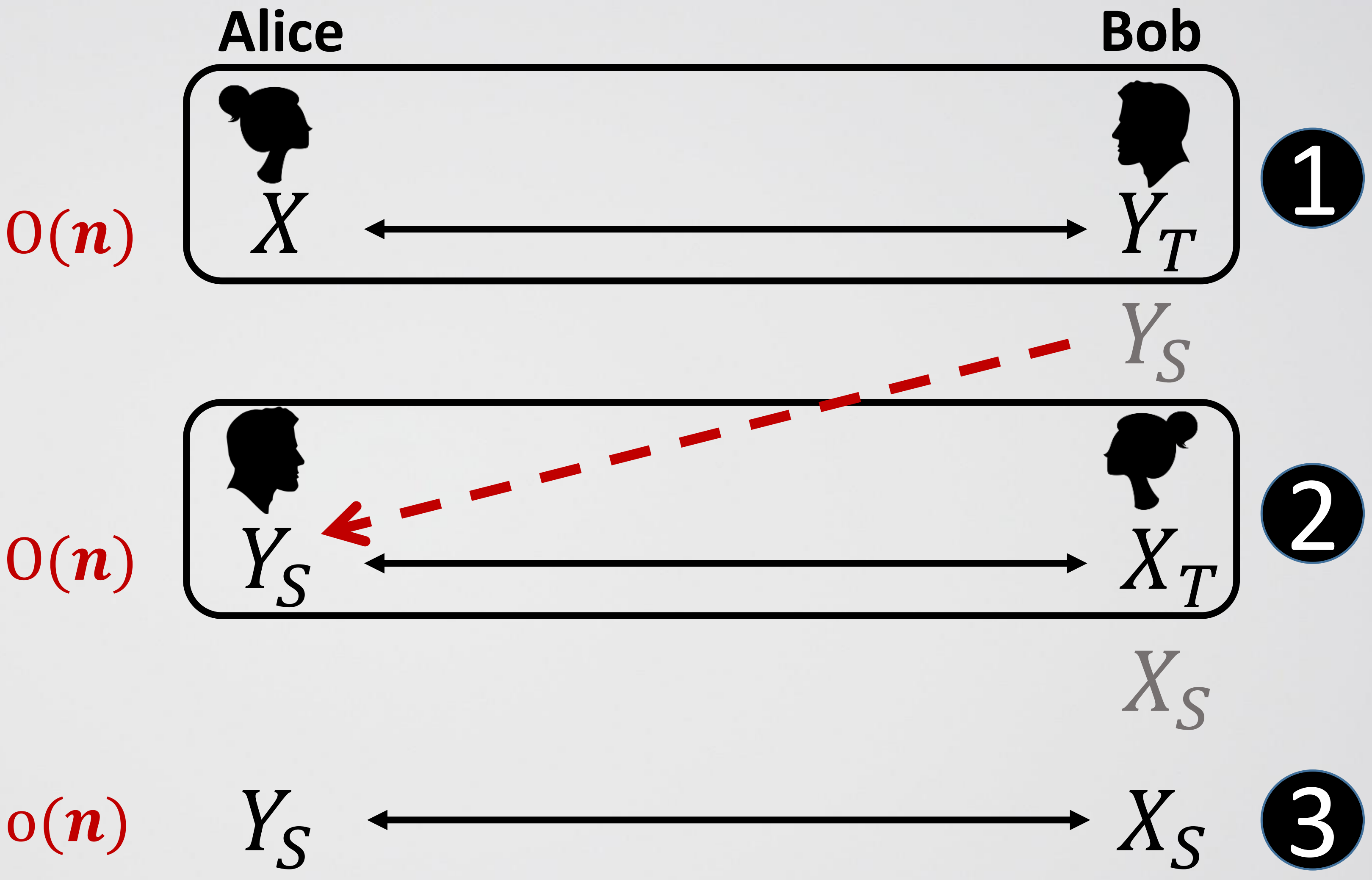


Dual Execution

3rd Step: Handling the Stash

- 3 phases protocol:



Experiments – PSI Analytics

vs. Previous Circuit-Based PSI [PSWW18]

$n = 2^{20}$ items of arbitrary bit-length

Fixing failure probability to 2^{-40}

	[HEK12]	**[PSWW18]**	**This work**
Communication	106 GB	25 GB	2.5 GB
Runtime (LAN)		5.5 min	2 min
Runtime (WAN)		25 min	4.5 min

vs. PSI-SUM [IKN+17]

65x faster (in LAN)

They leak intersection size

Conclusions

- MPC can help in getting rid of trusted parties
- Generic MPC is efficient if circuit size is small

- PSI in an important and interesting primitive, for which a naïve circuit is too large
- For such problems, need to design specific but adaptive MPC protocols